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Geometrical Aspects of Magnetic Monopoles

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The possible topological structures of elementary particles have been investigated to explore the possibility of the existence of magnetic monopoles. It is pointed out that when an elementary charged particle is depicted as an extended body such that the orientation of the internal space ("internal helicity") defines the fermion number, the global conservation of this does not allow the existence of a magnetic monopole. Again it is argued that when anisotropy is introduced in the microlocal space-time depicting the internal space of hadrons, this gives rise to the internal symmetry algebra and no non-Abelian gauge fields and Higgs scalars are necessary to have a grand unified scheme of interactions. This avoids the SU_2 and GUT monopoles. Besides, in this formalism, baryon number corresponds to the orientation or internal helicity of the composite system and the global conservation of this quantum number is found to be a consequence of Lorentz invariance. This forbids the existence of any sort of cosmological monopole in this Lorentz invariant Universe.

1. INTRODUCTION

It is well known that Dirac first suggested the existence of magnetic monopoles to have quantization of electric charge in an elegant way (Dirac, 1931). Later on, it was found out that non-Abelian gauge fields with spontaneous symmetry breakdown also suggests the existence of massive monopoles. In fact, t'Hooft (1974) and Polyakov (1974) have demonstrated that monopoles are an inevitable consequence of many gauge theories currently being used to unify electroweak and strong interactions. However, the experimental investigation in this direction gives a very poor demonstration of the fact that monopoles might exist. There is only a solitary uncorroborated candidate event from Stanford (Cabrera, 1982) which effectively contradicts the possible abundance of monopoles predicted from theoretical considerations. In view of this null experimental indication, we here propose to study the geometrical properties of monopoles and shall try to indicate

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the topological structure of elementary particles which forbids their existence in nature.

As we know, when an electrically charged point particle moves in the field of a magnetic monopole, a globally defined topology indicates the appearance of "strings." Again the motion of a Yang-Mills particle in the field of a non-Abelian monopole suggests that the internal symmetry like isospin becomes mixed up with ordinary angular momentum and the total angular momentum is defined as $\mathbf{J}=\mathbf{r}\times\mathbf{p}+\mathbf{T}$ where $\mathbf{T}$ is the isospin of the Yang-Mills field. Both these features may be associated with an anisotropy in the microlocal space-time. Indeed, in a recent paper (Banyopadhyay, unpublished, a) it has been shown that the anisotropic nature of the internal space-time may be taken to give rise to the internal symmetry of hadrons. This suggests that if the microlocal space-time structure admits a sort of anisotropy such as a "direction vector" attached to a world point, we can get features like magnetic monopoles without actually having these particles.

In this note we want to study the topological structure of a relativistic charged particle and shall show that the topology which determines the fermion number as a good quantum number does not allow the existence of a Dirac monopole. Again when an anisotropic feature is introduced in the internal space of hadrons so that a reflection group helps us to generate the internal symmetry (Bandyopadhay, unpublished, a) it is found that the baryon number corresponds to the orientation or " internal helicity" of the composite system and the global conservation of this quantum number is a consequence of Lorentz invariance. The existence of a monopole is forbidden by the global conservation of baryon number in such a topology.

In Section 2 we shall discuss briefly the role of reflection group and extended conformal symmetry in generating the internal symmetry of hadrons where the baryon number appears as the internal helicity of the composite system and for an elementary charged particle, lepton number appears as the internal helicity of an extended system. In Sections 3 and 4 we shall study the topological features of such systems and shall show that no monopoles appear in this scheme.

2. REFLECTION GROUP, EXTENDED CONFORMAL SYMMETRY, AND THE INTERNAL HELICITY OF A PARTICLE

Here we recapitulate certain features of reflection group, extended conformal symmetry and the internal helicity of particles. In a recent paper Budini (1979) has argued that we can generate an internal isospin algebra from the conformal reflection group. The simplest conformally covariant spinor field equation postulated as an $O(4,2)$ covariant equation in a pseudo-Euclidean manifold $M^{4,2}$ is of the form

$$\left(\Gamma_a \frac{\partial}{\partial \eta_a} + m\right) \mathscr{E}(\eta) = 0 \tag{1}$$

when the elements of the clifford algebra Γ_a are the basis unit vectors of $M^{4,2}$, m is a constant matrix, and $\mathscr{E}(\eta)$ is an eight-component spinor field. Cartan (1966) has shown that in the fundamental representation where the unit vectors are represented by the 8×8 matrices of the form

$$\Gamma_a = \begin{vmatrix} 0 & \Xi \\ H & 0 \end{vmatrix} \tag{2}$$

the conformal spinors $\mathscr{E}$ are of the form

$$\mathscr{E} = \begin{vmatrix} \varphi_1 \\ \varphi_2 \end{vmatrix} \tag{3}$$

where φ_1 and φ_2 are Cartan semispinors. The characteristic property of these spinors is that for any reflection φ_1 and φ_2 interchange. In this basis, equation (1) becomes equivalent in the Minkowski space to the coupled equations

$$i\partial\!\!\!/ \varphi_1 = -m\varphi_2$$
$$i\partial\!\!\!/ \varphi_2 = -m\varphi_1 \tag{9}$$

However, it is also possible to obtain from equation (1) a pair of standard Dirac equations in Minkowski space. To this end, we have to act on with a unitary transformation G given by

$$g = \begin{vmatrix} L & R \\ R & L \end{vmatrix} \tag{5}$$

where

$$L = \tfrac{1}{2}(1 + \gamma_5), \qquad R_2 = \tfrac{1}{2}(1 - \gamma_5), \qquad \text{and } \gamma_5 = \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix}$$

With this, we have

$$G\mathscr{E} = \mathscr{E}^D = \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix} \tag{6}$$

and

$$G^{-1}\Gamma_\mu G = \Gamma_\mu^D = \begin{vmatrix} \gamma_\mu & 0 \\ 0 & \gamma_\mu \end{vmatrix} \tag{7}$$

This suggests that equation (1) is equivalent in Minkowski space to the pair of standard Dirac equations

$$(i\partial\!\!\!/ + m)\varphi_1 = 0$$
$$(i\partial\!\!\!/ + m)\psi_2 = 0 \tag{8}$$

It is to be noted that space or time reflection interchanges φ_1 and φ_2 but transforms φ_1 and φ_2 into themselves. Conformal reflection (inverse radius transformation) interchanges both $\varphi_1 \rightleftharpoons \varphi_2$ and $\varphi_1 \rightleftharpoons \varphi_2$. It should be added that ψ_1 and ψ_2 may represent physical free massive fermions, whereas φ_1 and φ_2 do not unless they are massless, since may obey coupled equations.

Budini has suggested that conformal reflection algebra corresponding to a reflection group generates an internal symmetry algebra of hadrons in the sense that it commutes with the Poincaré Lie algebra. To study the conformal reflection algebra it is to be noted that since $O(3, 1)$ is a subgroup of $O(4, 2)$, the conformal reflection group will contain as a subgroup the Lorentz reflection group L_4 of four elements

$$L_4 = E, S, T, ST = \mathrm{J} \tag{9}$$

where E = identity, S = space reflection, T = time reflection and $ST = \mathrm{J}$ = strong reflection. In $M^{4,2}$ space if the coordinates are taken to be η_1, η_2, η_3, η_5, η_0, η_6 with the metric $(++++--)$ the reflection

$$s_5\colon \quad \eta_5 \to \eta_5' = -\eta_5$$
$$T_6\colon \quad \eta_6 \to \eta_6' = -\eta_6 \tag{10}$$

correspond in Minkowski space the inverse radius transformation and the same $\otimes\mathrm{J}$. With them we may build up the four-element Abelian group

$$C_{p_6} = E, S_5, T_6, S_5T_6 \tag{11}$$

which is called the partial conformal reflection group. Then the total conformal reflection group indicated by C_6 is given by the direct product

$$C_6 = C_{p_6} \otimes L_4 \tag{12}$$

The conformal reflection group is represented in conformal spinor space by the algebra $U_{4,C}$ which may be called the conformal reflection algebra.

Let $\mathscr{C}$ be a conformal spinor in the Dirac basis

$$\mathscr{C}^D = \begin{vmatrix} \psi_1 \\ \psi_2 \end{vmatrix}$$

where we know that the Lorentz reflection group L_4 when acting on the Dirac spinors ψ_i is isomorphic to a U_2 algebra whose Hermitian elements are given by the matrices 1, γ_0, $i\gamma_0\gamma_5$, γ_5. Now the transformations S_5, T_6,

S_5T_6 when acting on the Dirac doublet of the conformal spinor $\mathscr{E}^D$ will correspond

$$S_5 \rightarrow \Gamma_5^D$$
$$T_6 \rightarrow \Gamma_6^D$$
$$S_5T_6 \rightarrow \Gamma_5^D\Gamma_6^D \tag{13}$$

Thus the group given by C_{p_6} will be represented by the Lie algebra $U_{2,C}$ and the corresponding real subalgebra SU_2 may be obtained taking Hermitian elements $i\Gamma_5$, Γ_6, $\Gamma_5\Gamma_6$. Thus because of equation (12), the group C_6 is ismorphic to the product

$$U_{2,C} \otimes U_{2,C} = U_{4,C} \tag{14}$$

Then Budini with elegant arguments has proved the following propositions:

(1) The reflection algebra $U_{2,C}$ corresponding to the partial conformal reflection C_{p_6} is an internal symmetry algebra for the conformal spinor doublets. For massive (but degenerate) components of the doublet $U_{2,C}$ is maximal. For massless conformal spinors or for a system of massive conformal spinors interacting at very short distances, the direct product of the partial conformal reflection group times the strong reflection in Minkowski space generates a restricted internal symmetry algebra of order eight which can be put in the form $U_{2C,L} \oplus U_{2C,R}$. This $U_{2C,L} \oplus U_{2C,R}$ algebra may be reduced to two independent SU_2 algebras represented by the eight four-dimensional matrices $L \times \sigma_\mu$, $R \times \sigma_\nu$ ($[L = \frac{1}{2}(1 + \gamma_5)$, $R = \frac{1}{2}(1 - \gamma_5))]$ acting on the two independent doublets of Weyl fields into which the massless conformal spinor or the system of interacting massive spinors at short distances splits.

This reflection algebra which generates the internal symmetry of hadrons necessarily indicates the existence of a direction vector in the microlocal space-time. This geometrical formalism also demands the existence of a disconnected gauge group to have a proper understanding of the symmetry of elementary particle interactions. Indeed, in a recent paper have argued that when conformal spinors are taken to be the constituents of hadrons which split into two Cartan semispinors in Minkowski space giving rise to particles and antiparticles, the extended conformal group which includes apart from conformal transformations, space, time, and conformal reflection appears to be the grand unified group of all interactions. This can nicely explain why we have only four types of interactions in nature and avoid the introduction of all fictitious particles like gauge bosons, Higgs scalars, and gluons. In this scheme of the geometrical origin of internal symmetry, mesonic configurations are distinguished from baryonic configurations by the fact that in the former case two constituents appear with handedness opposite to each other and as such there is no intrinsic handedness or orientation which is manifested outside the configuration but for baryons constituents appear with a specific handedness such that this particular orientation is related with the baryon number (Bandyopadhyay, unpublished, a). However this "direction vector" or internal handedness does not destroy Lorentz invariance. This follows from the fact that in this scheme, since particles and antiparticles appear with equal footing just with opposite handedness this demands invariance under CP and CPT transformations and Lorentz invariance is manifested here through CPT invariance.

From this analysis, it appears that baryon number is a manifestation of the intrinsic handedness of a composite system. Similarly, lepton number can also be depicted as a manifestation of the intrinsic handedness of an extended object. Indeed an extended object can be depicted by a wave function of the form $\psi(\lambda_\mu, \xi_\mu)$, where ξ_μ is an internal variable attached to each space-time point. The introduction of this variable gives rise to dilatation in addition to Poincaré transformation and thus gives rise to an extension of the internal space. When this ξ_μ is identified with a "direction vector" or "velocity vector" this may be taken to correspond to an internal helicity which may be related with the lepton number. The relevance of conformal symmetry in this case appears through the fact that two such four-component spinors with opposite internal helicity may be considered as an eight-component conformal spinor and as such the group properties of such an extended body should be considered from the viewpoint of its possible extension to the conformal symmetry group $O(4,2)$.

Thus the anisotropic features of microlocal space-time giving rise to the internal symmetry of hadrons as well as lepton number for an extended object helps us to establish an interlink between the discrete symmetry like CPT invariance and the continuous symmetry like Lorentz invariance through the geometrical properties of particles. The particle antiparticle configurations generated by $SO(4,2)$ eight-component spinors such that the doublet of Cartan semispinors with opposite internal handedness represent particle and antiparticle state or vice versa help us to split the group

$$SO(4,2) = SO(3,1)_L \otimes SO(3,1)_R \tag{15}$$

which indicates that the left-handed and right-handed systems correspond to these configurations. This suggests that we will have two independent Poincaré invariant systems distinguished by their intrinsic handedness. The physical implications of this feature are given by the fact that if we take the conventional definition of particles and antiparticles the sense of right and left can be determined by any parity-violating interaction. On the other hand, if we take the conventional sense of left and right, particles and antiparticles can be distinguished. Thus right and left are still indiscernible and the isotropy of space is restored in the external world.

This internal helicity of a hadron or of an extended object gives rise to certain unique topological features of particle configurations which forbid the existence of magnetic monopoles in this Lorentz invariant universe. Indeed the conservation of lepton number or baryon number is in contradiction with the existence of Dirac or grand unified monopoles when these quantum numbers are associated with the internal helicity as discussed above.

3. TOPOLOGICAL STRUCTURE OF AN ELEMENTARY CHARGE AND MAGNETIC MONOPOLE

In a recent paper (Bandopadhyay, unpublished, c) it has been argued that the non-Hermitian character of the position operator of a relativistic particle has something to do with the topological structure of the particle. Indeed as Kalnay and Toledo (1967) have pointed out the non-Hermitian character of an operator gives rise to a complex eigenvalue and as a complex number can be considered as an ordered pair of real numbers, this feature may be associated with an extended nature of a relativistic particle. It has been argued that this extended body helps us to define an internal helicity or orientation which may be related with the fermion number and the second factor in the complex eigenvalue corresponds to this number which is a constant of motion.

An extended body can be depicted by the de Sitter group $SO(4, 1)$. Indeed, the wave function of the form $\psi(\lambda_\mu, \xi_\mu)$ where ξ_μ is an "internal variable" attached to each point in space-time gives rise to dilatation in addition to Poincaré transformation and thus may be taken to depict an extension of the internal space. When ξ_μ is identified with a direction vector or velocity vector this may be taken to correspond to an "internal helicity." Roman et al. (1972) have shown that the inclusion of this extra variable extends the Poincaré group $SO(3, 1)$ to the de Sitter group $SO(4, 1)$.

The irreducible representation of $SO(4)$ are characterized by two numbers (k_0, n), where k_0 is integer or half-integer and n is a natural number. These two numbers are related to the values of the Casimir operators by

$$\tfrac{1}{2}s_{\alpha\beta}s^{\alpha\beta} = k_0^2 + (|k_0| + n)^2 - 1$$

$$\tfrac{1}{8}\varepsilon^{\alpha\beta\gamma\delta}s_{\alpha\beta}s_{\gamma\delta} = k_0(|k_0| + n) \tag{16}$$

where $s_{\alpha\beta}$ $\alpha, \beta = 1, 2, 3, 4$ are the generators of the group. The irreducible representations of $SO(4, 1)$ have been investigated by Dixmier (1961). Barut and Bohm (1970) have shown that the representations of $SO(4, 1)$ given by

$k_0=+\frac{1}{2}$ and $-\frac{1}{2}$ can be extended to two inequivalent representations of $SO(4,2)$. In fact, these k_0 values actually correspond to the eigenvalues of the operator $K_0=\frac{1}{2}(a^+a-b^+b)$ in the oscillator representation of the $SO(3)^1\otimes SO(3)^2$ basis of $SO(4,1)$. Barut and Bohm have shown that no other representations excepting those corresponding to the eigenvalues $k_0=+\frac{1}{2}$ and $-\frac{1}{2}$ apart from the trivial case $k_0=0$ can be fully extended to $SO(4,2)$. Besides, the value of k_0 as well as its signature is an $SO(4,2)$ invariant.

The representation ($s=0$, $k_0=0$) in the conformal interpretation of $O(4,2)$ describes massless spin-0 particles. The representation $s=\frac{1}{2}$, $k_0=\pm\frac{1}{2}$ describes the helicity states of a massless spinor. Now for a massive particle, the conformal invariance breaks down so that $k_0=\pm\frac{1}{2}$ cannot be interpreted as helicity states in the conventional sense. To find the relevance of these states for a massive particle, we note that an $O(4,2)$ spinor is given by an eight-component spinor which may be split into two four-component spinors with a certain "orientation." Indeed, the doublet of four-component spinors $\left|{\varphi_1 \atop \varphi_2}\right|$ representing an $O(4,2)$ spinor $\mathscr{E}$ is characterized by the fact that the space, time, or conformal reflection changes $\varphi_1\rightleftharpoons\varphi_2$. Besides in Minkowski space, they satisfy the coupled equations (4). Now it is noted that the coupled equations (4) do not allow the Cartan semispinor doublets $\left|{\varphi_1 \atop \varphi_2}\right|$ to be physically observable unless $m=0$. However, if we define φ_1 and φ_2 such that they represent two different internal helicity states given by $k_0=+\frac{1}{2}$ and $-\frac{1}{2}$, i.e., $\varphi_1=\psi(k_0=+\frac{1}{2})$ and $\varphi_2=\psi(k_0=-\frac{1}{2})$ the equations (4) can be reduced to a single equation with two internal degrees of freedom when the linear combination of $\psi(k_0=-\frac{1}{2})$ and $\psi(k_0=+\frac{1}{2})$ represents an eigenstate. Now to retain the four-component nature of the spinor in Minkowski space, these two internal degrees of freedom should be associated with particle-antiparticle states. Evidently this property of φ_1 and φ_2 satisfies the criteria that space, time, or conformal reflection changes one into the other. This follows from the fact that the parity operator changes the sign of k_0. Besides as φ_1 and φ_2 are related here to particle-antiparticle states, the T operator also changes $\varphi_1\rightleftharpoons\varphi_2$. Again considering the Iwasawa decomposition KAN of the de Sitter group $SO(4,1)$ where K is the group $SO(4)$, the maximal compact subgroup of $SO(4,1)$, A the group $SO(1,1)$ generated by D the dilatation operator ($D=M_{54}$) and N is a nilpotent Abelian group generated by K_i, the special conformal transformation ($K_i=M_{i5}+M_{i4}$, $i=1,2,3$), it appears that the conformal reflection ($\eta_4\to\eta_4'=-\eta_4$, $\eta_5\to\eta_5'=-\eta_5$) will also change $\varphi_1\rightleftharpoons\varphi_2$ when the internal helicity $K_0=+\frac{1}{2}$ and $-\frac{1}{2}$ are taken to correspond to the states φ_1 and φ_2, respectively. Thus the doublet of massive spinors with spatial extension of their structures with the above properties can represent an $O(4,2)$ spinor. In view of this we can avoid the other representations of the operator $K_0=\frac{1}{2}(a^+a-b^+b)$

of the $SO(4,1)$ group excepting those with eigenvalues $k_0 = \pm\frac{1}{2}$ as these are the only eigenvalues which can be fully extended to $SO(4,2)$ and remain irreducible under $SO(4,1)$. Thus the internal helicity states can be related with the fermion number (lepton number) of the massive particle when described as an extended body as this can take the unique values $k_0 = +\frac{1}{2}$ and $-\frac{1}{2}$ only. It is noted that no boson can be described in this way and this picture suggests that bosons must be composite objects.

This description of a charged and massive fermion as an extended object finds its relevance in earlier papers (Bandyopachyay, 1973, 1974) where it is shown that the charge and mass of an elementary fermion (lepton) can be taken to be of dynamical origin. There it has been shown that a charged lepton like $e^-(\mu^-)$ can be represented by $(\nu_e s)$ $(\nu_\mu s)$ where s is the system of photons interacting weakly with the two-component massless $\nu_e(\nu_\mu)$ at n space-time points within a quantized domain and these nonlocal interactions can give rise to the charge and mass of an $e^-(\mu^-)$ as well as two more components necessary for a four-component spinor. The two more components arise here from the form factor which describes the internal spatial extension with opposite orientation which gives rise to particle and antiparticle states. These internal orientations actually correspond to the two k_0 values depicting fermion number. In this picture the quantization of charge is related with the quantization of space-time.

To find the topological aspects of such an extended object given by $SO(4,1)$ group structure, we here find out the localization region of such a relativistic particle using Mackey's theory of imprimitivity. In this connection we briefly mention two main results of the Mackey theory (Mackey, 1952, 1953, 1958).

Let H be a closed subgroup of the lie group $\mathscr{G}$ (countable at infinity), L unitary representation of H on Hilbert space $\mathscr{H}$, U^L the corresponding unitarily induced representation of g and let g/H be the left coset space considered as a g-transformation space. Then every unitarily induced representation U^L of g gives rise to a canonical system of imprimitivity E^L defined by

$$(E^L(\Delta)f)(x) = \begin{cases} f(x) & \text{if } xH \in \Delta;\ x \in g \\ 0 & \text{if } xH \notin \Delta;\ f \in H^L \end{cases} \tag{17}$$

Mackey's imprimitivity theory states.

If there exists a transitive system of imprimitivity E for a given representation U of g based on g/H then there is a unitary representation L of H unique to within unitary equivalence such that E is unitarily equivalent to the canonical system of imprimitivity for U^L.

This suggests that there is no system of imprimitivity based on g/H other than the canonical one and it exists whenever the considered representation U of g appears to be unitarily induced by a unitary representation of H. Thus a particle is localizable in $\Delta \subset g/H$ if the corresponding representation U of g is unitarily induced by a unitary representation of H.

In our approach a relativistic charged and massive particle with spatial extension is described by the unitary irreducible representation U^L of the de Sitter group $SO(4, 1)$. If there exists a subgroup H of $SO(4, 1)$ such that the obtained representation of $SO(4, 1)$ appears to be unitarily induced by a unitary representation of H, the corresponding particle is localized in a region $\lhd$ of the space $SO(4, 1)/H$. As the subgroup H we take the group MAN where M is the group $SO(3)$, A the group $SO(1, 1)$ generated by dilatation D, and N is a nilpotent Abelian group generated by k_i as discussed above. Now considering the Iwasawa decomposition of as KAN, K being the group $SO(4)$, we find that the extended charged and massive particle yields the topology of a 3-sphere

$$\frac{SO(4, 1)}{MAN} \simeq K/M \simeq S^3 \tag{18}$$

It is to be noted that the same topology of localization region has been derived by Bayen and Niederle (1981) for a massless particle using the conformal interpretation of such particles.

Now for the gauge field of a charged particle, we can construct a 2-form F defined by $F = \frac{1}{2}F_{\mu\nu}\, dx^\mu\, dx^\nu$, where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Because of the fact that $\oint_{\partial c^3} F = 0$, where c^3 is a 3-chain of S^3 and ∂c^3 is its boundary, according to de Rahm's theorem we find that there exists a smooth 1-form $A = A_\mu\, dx^\mu$ on the Minkowski space M such that $F = dA$. This happens because if we extend M nontrivially to $B = R^2 \times S^3$ we have the vanishing second Betti number.

This suggests that this topological structure of an elementary charged particle does not allow a magnetic monopole to exist. Because in case a monopole exists, we would have $\oint F = 4\pi g \neq 0$ and there should not be any smooth choice of A such that $dA = F$. However, this topological structure allows the existence of an electric charge. Indeed S^3 can be obtained from the three dimensional Euclidean space R^3 by adding a pair of antipodal points. Now noting that we have arrived at the group structure of the charged particle $SO(4, 1)$ by attaching a direction vector ξ_μ to the space-time point x_μ and this eventually gives rise to the internal helicity $k_0 = \pm\frac{1}{2}$ corresponding to the fermion number, we can identify the pair of points of S^3, $x = 0, 0, \pm 1, 0$ representing two opposite internal helicities. By cutting out this pair of antipodal points, we get the pierced sphere $S^2 \times S^1$. Now taking c^2 as a 2-chain of S^2 and c^1 a 1-chain of S^1 and defining the one form $\xi = \xi_\mu\, d\xi^\mu$

corresponding to the direction vector we find that within the localization region S^3 the decomposition $S^3 \to S^2 \times S^1$ leads to

$$\oint_{c^2} F \otimes \oint_{c^1} \mathscr{C} \neq 0$$

This is due to the fact that the second Betti number does not vanish in the case $\oint_{c^2} F$ and the curve c^1 has no boundary ($\partial c^1 = 0$). Indeed, two opposite orientations of c^1 leads to two opposite internal helicities in this picture. Thus we find that within the localization region S^3 of the particle, there can be an extended body with charge having opposite internal helicities, but the condition $\oint_{\partial c^3} F = 0$ suggests that there is zero net flux through every surface ∂c^3 that is closed ($\partial\partial c^3 = 0$) i.e.,

$$\oint_{\partial c^3} F = \oint_{\partial c^3} dA = \oint_{\partial\partial c^3} A = 0$$

implying the existence of a smooth vector potential A_μ which forbids the existence of a magnetic monopole. Thus the very topology of an extended charged particle such that the internal orientation gives rise to the fermion number does not allow a magnetic monopole to exist.

4. NON-ABELIAN GAUGE THEORIES AND MONOPOLES

As we have mentioned earlier, non-Abelian gauge theories give rise to heavy monopoles. Polyakov and t'Hooft independently showed that monopoles appear as stable solutions of the spontaneously broken Yang–Mills field equations and were required by a large class of such theories. When the grand unified group SU_5 is sponaneously broken to U_1, it gives rise to superheavy monopoles (GUM) having mass $> 10^{14}$ GeV. In a GUT model, there is no difference between strong, weak, and electromagnetic interaction at a high temperature. This temperature is typically of the order of 10^{15} GeV. As the temperature is lowered we can have a nonzero value of the vaccum expectation value of the scalar Higgs field. This spontaneous breaking of the symmetry can be frozen in a fixed space direction. In cosmology this direction can only span a distance equal to the velocity of light times the age of the universe. At the early universe, these domains were very small. A GUT monopole can be viewed as the coalescence of several of these domains. The monopole is heavy because the domains are confined in a space of the order of 10^{-4} cm.

In case the internal symmetry of hadrons arises from the microlocal space-time structure then of course it is not necessary to take into account non-Abelian fields to exist. Indeed, in Bandyopadhyay (unpublished, a) it has been shown that the internal space of hadrons is anisotropic in nature so that a particular direction (z axis) is fixed in such a way that positive z axis and negative z axis corresponds to particle and antiparticle configurations or vice-versa. A reflection group can then be taken to generate the internal symmetry of hadrons. Lorentz invariance is maintained in the external space as CPT invariance is inbuilt in this mechanism. This result can be achieved in a very elegant way when eight-component conformal spinors are taken to be the constituents of hadrons and these split into two Cartan semispinors in Minkowski space giving rise to particle and antiparticle configurations. The internal symmetry of hadrons arises from the discrete symmetry of conformal reflection and strong reflection which gives rise to two independent algebras $SU_{2L} \oplus SU_{2R}$ corresponding to the isospin symmetry of particle and antiparticle configurations. Hypercharge is here related with the fixed l_z values and baryon number depicts the internal helicity or orientation of the composite system (Bandyopadhyay, unpublished, a) As mentioned in Section 2, in this scheme extended conformal group appears to be the grand unified group and various combinations of internal symmetry patterns with CP invariance give rise to only four types of interactions, viz., strong, weak, electromagnetic and gravitational interactions, in a unique way (Bandyopadhyay, 1984). However, for these processes no fictitious particles like gauge bosons, Higgs scalars, and gluons are necessary.

An interesting feature of this scheme is that baryon number, conserving processes are forbidden just by the requirement of Lorentz invariance. This is due to the fact that as the baryon number is given by the internal helicity or orientation of the internal space, the destruction of this orientation in any process will also violate the particle–antiparticle symmetry. This means that CPT invariance and as such Lorentz invariance will be violated in such processes.

Since the internal symmetry is here linked up with the discrete symmetry like conformal and strong reflection, for electromagnetic interaction we will have to consider the disconnected gauge group $U_{1L} \otimes U_{1R} = U_1 \otimes \{1, d\}$, where d is the orientation reversing operation. Kiskis (1978) has studied this feature in details and has argued that for a simply connected space, this poses no problem related to global charge conservation, though for nonsimply connected space global violation of charge conservation may occur.

From the above analysis, it appears that for the grand unified theory where the internal symmetry is taken to be a property of the microlocal space time, we have no necessity to consider the existence of gauge bosons and Higgs sclars and as such no SU_2 or GUT monopoles can appear. Moreover, if we just take into account that electromagnetic gauge group is given by a disconnected gauge group where the potential is known up to the sign and baryon number violating processes are forbidden by Lorentz invariance no cosmological monopole of any sort can exist in nature. Indeed, if such monopoles exist, fermions may change their nature when scattered off in the s wave. This is due to the fact that a magnetic charge h has an associated angular momentum

$$J = \frac{gh}{4\pi} a$$

This acquires a sign change when the fermion passes through the monopole core because $a \to -a$. Angular momentum can only be conserved if there is a simultaneous change of sign $g \to -g$. This requires the transition of the fermion into antifermion. But the disconnected gauge group does not allow such a transition in a continuous process and also the forbiddeness of baryon number nonconservation by Lorentz invariance does not allow this to occur.

5. DISCUSSION

We have shown above that the topological structure of an elementary charged particle as well as of hadrons may be such that this does not allow magnetic monopoles to exist. In case of a Dirac particle if we associate fermion number with the internal helicity of the extended body of the charged particle, the localization region is found to be a 3-sphere S^3 and so the cyclic integrals $\oint_{\partial c^3} F$ vanish indicating that there is no monopole. A consequence of this is that the fermion number will be globally conserved.

For SU_2 and GUT monopoles we have observed that if the internal symmetry of hadrons is generated from the reflection group, no gauge bosons and Higgs scalars are necessary to have the grand unification of all interactions, and as such no such monopoles appear in this formalism. Moreover, if we take into account that baryon number corresponds to the internal helicity or orientation of the composite system and electromagnetic gauge group appears as a disconnected gauge group $U_{1L} \otimes U_{1R} = U_1 \otimes \{1, d\}$, where d is the orientation reversing operation, the global conservation of baryon number is found to be a consequence of Lorentz invariance. This suggests that no cosmological monopoles of any kind can exist as the interaction of these entities with a baryon may change it to an antibaryon, which we have argued cannot occur in a Lorentz invariant way. This suggests that in this Lorentz invariant Universe, there cannot be any monopole.

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